University of California, Santa Barbara

Department of Electrical and Computer Engineering

ECE 152A – Digital Design Principles

Homework #1
Solution

Problem #1:
Demonstrate by means of truth tables the validity of the following identities:

1. DeMorgan's theorem for three variables: *(xyz)' = x' + y' + z'*

x	y	z	xyz	(xyz)'	x'	y'	z'	x' + y' + z'
0	0	0	0	**1**	1	1	1	**1**
0	0	1	0	**1**	1	1	0	**1**
0	1	0	0	**1**	1	0	1	**1**
0	1	1	0	**1**	1	0	0	**1**
1	0	0	0	**1**	0	1	1	**1**
1	0	1	0	**1**	0	1	0	**1**
1	1	0	0	**1**	0	0	1	**1**
1	1	1	1	**0**	0	0	0	**0**

2. The second distributive law: *x + yz = (x + y)(x + z)*

x	*y*	*z*	*yz*	*x + yz*	*(x + y)*	*(x + z)*	*(x + y)(x + z)*
0	0	0	0	**0**	0	0	**0**
0	0	1	0	**0**	0	1	**0**
0	1	0	0	**0**	1	0	**0**
0	1	1	1	**1**	1	1	**1**
1	0	0	0	**1**	1	1	**1**
1	0	1	0	**1**	1	1	**1**
1	1	0	0	**1**	1	1	**1**
1	1	1	1	**1**	1	1	**1**

3. The consensus theorem: $xy + x'z + yz = xy + x'z$

x	y	z	xy	$x'z$	yz	$xy + x'z + yz$	$xy + x'z$
0	0	0	0	0	0	**0**	**0**
0	0	1	0	1	0	**1**	**1**
0	1	0	0	0	0	**0**	**0**
0	1	1	0	1	1	**1**	**1**
1	0	0	0	0	0	**0**	**0**
1	0	1	0	0	0	**0**	**0**
1	1	0	1	0	0	**1**	**1**
1	1	1	1	0	1	**1**	**1**

Problem #2:
Simplify the following Boolean expressions to a minimum number of literals:

1. $x'y' + xy + x'y$

$x'(y' + y) + xy$

$x' + xy$

$(x' + x)(x' + y)$

$\mathbf{= x' + y}$

2. $(x + y)(x + y')$

$(x + y)$
$\underline{(x + y')\ \{\text{ multiply out }\}}$
$xx + xy + xy' + yy'$

$x + xy + xy'$

$\mathbf{= x}$

3. $x'y + xy' + xy + x'y'$

$(x' + x)y + (x' + x)y'$

$y + y'$

$\mathbf{= 1}$

4. $x' + xy + xz' + xy'z'$

$x' + xy + \quad xz'$

$x' + x(y + z')$

$\mathbf{= x' + y + z'}$ { by $A + A'B = A + B$ }

5. $xy' + y'z' + x'z'$

$\mathbf{= xy' + x'z'}$ { *by consensus theorem* }

Problem #3:
Simplify the following Boolean expressions to a minimum number of literals:

1. $ABC + A'B + ABC'$

   $AB(C + C') + A'B$

   $AB + A'B$

   $(A + A')B$

   $\mathbf{= B}$

2. $x'yz + xz$

   $z (x'y + x)$

   $\mathbf{= z (y + x)}$

3. *(x + y)' (x' + y')*

    x'y' (x' + y')

    x'y'x' + x'y'y'

    = x'y'

4. *xy + x(wz + wz')*

    xy + x(w(z + z')

    xy + xw

    = x (y + w)

5. *(BC' + A'D)(AB' + CD')*

    <u>*(AB' + CD')   { multiply out }*</u>

    A(BB')C' + (AA')B'D + B(CC')D' + A'C(DD')

    = 0

Problem #4:
Reduce the following Boolean expressions to the indicated number of literals:

1. *A'C' + ABC + AC'*   to three literals

    C' (A' + A) + ABC

    C' + ABC

    = AB + C'

2. *(x'y' + z)' + z + xy + wz*   to three literals

    (x + y) z' + z + xy   *{ z + wz = z }*

    (x + y) + z + xy   *{ z + z'( ) = z + ( ) }*

    = x + y + z

3. *A'B(D' + C'D) + B(A + A'CD)* to one literal

   A'BD' + A'BC'D + AB + A'BCD

   A'BD (C + C') + A'BD' + AB

   A'B (D + D') + AB

   A'B + AB

   = B

4. *(A' + C)(A' + C')(A + B + C'D)* to four (or fewer?) literals

   <u>*A' + C' { multiply out }*</u>

   (A'A' + A'C + A'C' + CC') (A + B + C'D)

   A' (A + B + C'D)

   = A'(B + C'D)

Problem #5:
Find the complement of F = x + yz; then show that F (F') = 0 and F + F' = 1;

F' = (x + yz) ' = (x') (y' + z') ***= x'y' + x'z'***

F (F') = 0 =

(x + yz) (x'y' + x'z')

xx'y' + xx'z' + x'yy'z + x'yzz'

= 0

F + F' = 1 =

x + yz + x'y' + x'z'

x + yz + y' + x'z'

x + yz + y' + z'

x + y' + y + z'

= 1

Problem #6:
Find the complement of the following expressions:

1. $F = xy' + x'y$

   $\mathbf{F' = (x' + y)(x + y') = xy + x'y'}$

2. $F = (AB' + C)D' + E$

   $= AB'D' + CD' + E$

   $\mathbf{F' = (A' + B + D)(C' + D)E'}$

3. $F = AB(C'D + CD') + A'B'(C' + D)(C + D')$

   $ABC'D + ABCD' + A'B(CD + C'D')$

   $= ABC'D + ABCD' + A'BCD + A'BC'D'$

   $\mathbf{F' = (A' + B' + C + D')(A' + B' + C' + D)}$
   $\mathbf{(A + B' + C' + D')(A + B' + C + D)}$

4. $(x + y' + z)(x' + z')(x + y)$

   $\mathbf{= x'yz' + xz + x'y'}$

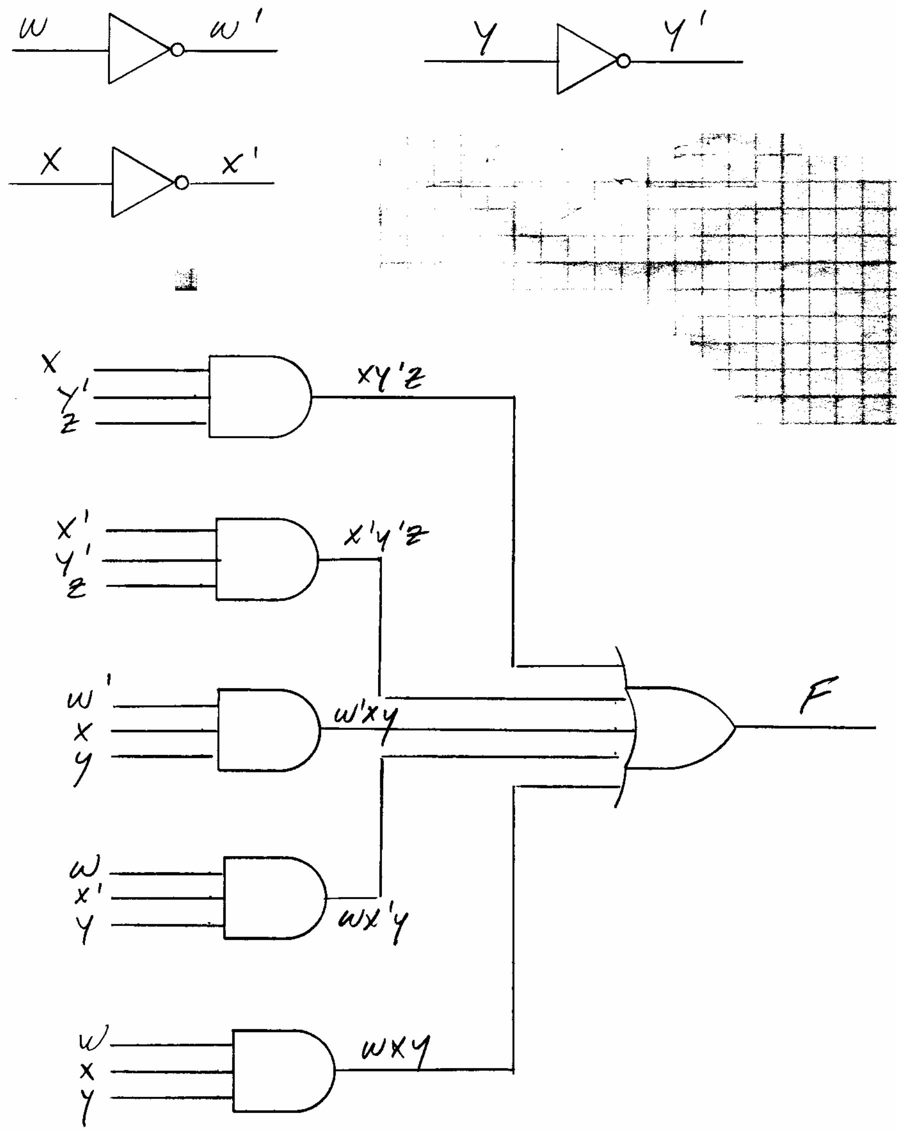
Problem #7:
Given the following Boolean function:

$$F = xy'z + x'y'z + w'xy + wx'y + wxy$$

1. Obtain the truth table for the function

	w	*x*	*y*	*z*	*xy'z*	*x'y'z*	*w'xy*	*wx'y*	*wxy*	*F=y'z+yw+xy*
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	1	0	0	0	1
2	0	0	1	0	0	0	0	0	0	0
3	0	0	1	1	0	0	0	0	0	0
4	0	1	0	0	0	0	0	0	0	0
5	0	1	0	1	1	0	0	0	0	1
6	0	1	1	0	0	0	1	0	0	1
7	0	1	1	1	0	0	1	0	0	1
8	1	0	0	0	0	0	0	0	0	0
9	1	0	0	1	0	1	0	0	0	1
10	1	0	1	0	0	0	0	1	0	1
11	1	0	1	1	0	0	0	1	0	1
12	1	1	0	0	0	0	0	0	0	0
13	1	1	0	1	1	0	0	0	0	1
14	1	1	1	0	0	0	0	0	1	1
15	1	1	1	1	0	0	0	0	1	1

2. Draw the logic diagram using the original Boolean expression



3. Simplify the function to a minimum number of literals using Boolean algebra

   xy'z + x'y'z + w'xy + wx'y + wxy

   (x + x') y'z + (w' + w) xy + wx'y

   y'z + xy + wx'y

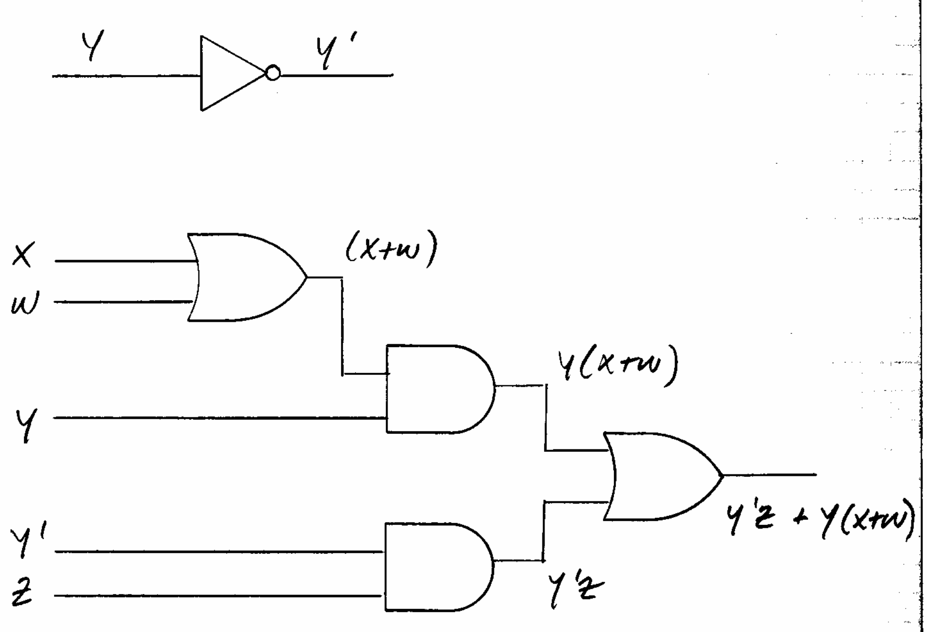
   y'z + y (x + x'w)

   = y'z + y (x + w)

4. Obtain the truth table of the function from the simplified expression and show that it is the same as the one in part 1

   See truth table above

5. Draw the logic diagram from the simplified expression and compare the total number of gates with the diagram of part 2



Problem #8:
Convert the following expressions into sum of products and product of sums:

1. *(AB + C) (B + C'D)*

   = AB + BC (SOP)

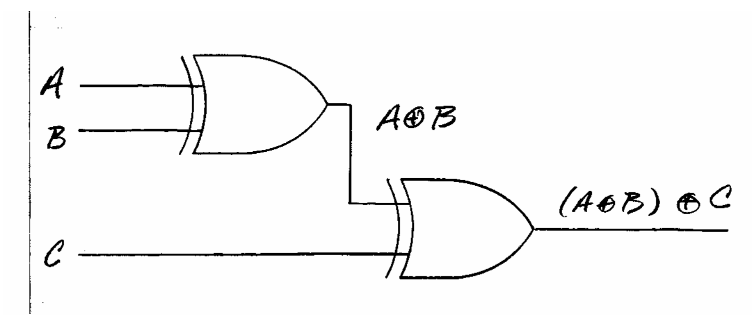
   = B(A + C) (POS)

2. *x' + x (x + y') (y + z')*

   = x' + y + z' (SOP)

   = x' + y + z' (POS)

Problem #9:
A 3-input exclusive OR gate can be constructed from 2, 2-input gates as shown below:



1. Generate the truth table for the 3 input XOR gate

A	B	C	A ⊕ B	⊕ C
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

2. Generate the Boolean expression for the 3 input XOR gate

   $$\mathbf{A \oplus B \oplus C = A'B'C + A'BC' + AB'C' + ABC}$$

Problem #10:
Show (using Boolean algebra) that the dual of the 2-input exclusive-OR function is equal to its complement.

XOR = $x'y + xy'$

Dual (XOR) = $(x' + y)(x + y') = x'x + x'y' + xy + yy'$

$$\boldsymbol{= xy + x'y'}$$

Complement (XOR) = $(x'y + xy')' = (x + y')(x' + y)$

$$= x'x + x'y' + xy + yy'$$

$$\boldsymbol{= xy + x'y'}$$

Problem #11:

1) $(xy+z)(y+xz)$

$$\begin{array}{l} (xy+z) \\ \underline{(y+xz)} \end{array}$$

$$xyy + yz + xxyz + xzz$$

$$xy + yz + xyz + xz$$

$$= xyz + xyz' + x'yz + xy'z$$

$$= m_7 + m_6 + m_3 + m_5$$

x	y	z	$(xy+z)(y+xz)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Problem #11 (cont):

$$= \Sigma m(3,5,6,7)$$

$$= \Pi M(0,1,2,4)$$

1.2) $(A'+B)(B'+C)$

$$A'+B$$
$$B'+C$$

$$A'B' + \cancel{BB'} + A'C + BC$$

EXPAND ALGEBRAICALLY:

$$A'B'C + A'B'C' + A'BC + ABC$$

$$= m_1 + m_0 + m_3 + m_7$$

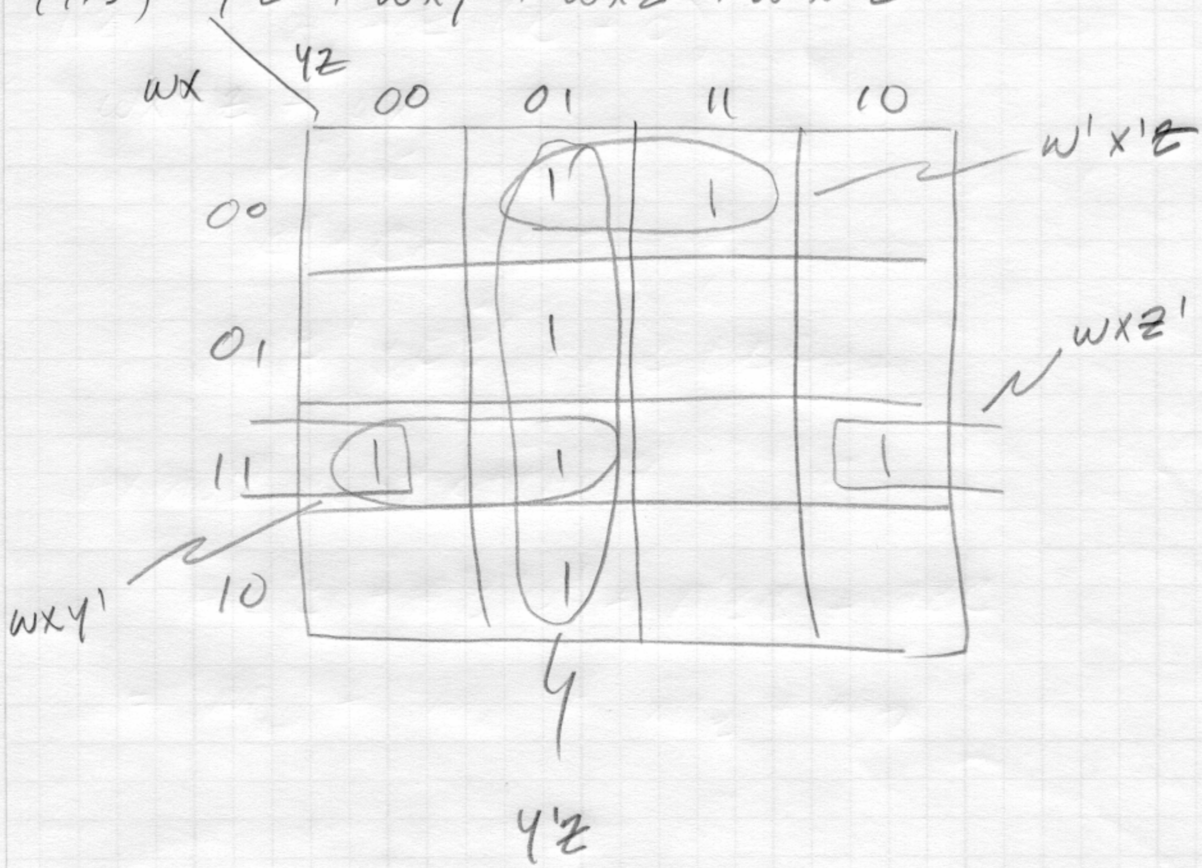
A	B	C	(A'+B)(B'+C)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Problem #11 (cont):

$$= \sum m(0,1,3,7)$$

$$= \prod M(2,4,5,6)$$

(1.3) $y'z + wxy' + wxz' + w'x'z$



$$= \sum m(1,3,5,9,12,13,14)$$

$$= \prod M(0,2,4,6,7,8,10,11,15)$$

Problem #12:

2.1) MINTERMS: m_2, m_3, m_6, m_7

$(x'yz', x'yz, xyz', xyz)$

2.2) MINTERMS OF F'

m_0, m_1, m_4, m_5

$(x'y'z', x'y'z, xy'z', xy'z)$

.3) SUM OF MINTERMS:

$= \Sigma m(2,3,6,7)$

.4) SIMPLIFY FUNCTION

$= x'yz' + x'yz + xyz' + xyz$

$= y(x'z' + x'z + xz' + xz)$

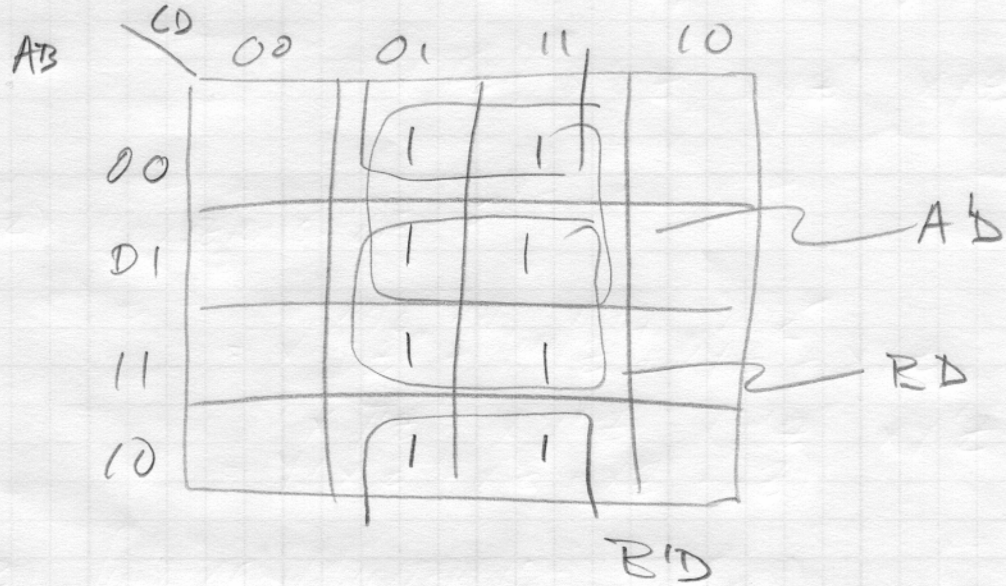
$= y(x'(z'+z) + x(z'+z))$

$= y(x'+x)$

$= \underline{\underline{y}}$

Problem #13:

.1) $B'D + A'D + BD$



$= D$

$= \sum m(1,3,5,7,9,11,13,15)$

$= \prod M(0,2,4,6,8,10,12,14)$

Problem #13 (cont):

2) $(xy+z)(xz+y)$

$$\begin{array}{r} xy+z \\ xz+y \\ \hline xyz + xz + xy + yz \end{array}$$

$x \backslash yz$	00	01	11	10
0			1	
1		1	1	1

yz

xy

xz

$= \sum m(3,5,6,7)$

$= \prod M(0,1,2,4)$

Problem #14:

1) $\sum(0,2,6,11,13,14)$

$$F' = \sum(1,3,4,5,7,8,9,10,12,15)$$

2) $\prod(0,3,6,7)$

$$F' = \sum(0,3,6,7)$$

Problem #15:

1) $\Sigma(0,1,5,7)$

$x \backslash yz$	00	01	11	10
0	1	1		
1		1	1	

$= x'y' + xz$

Problem #15 (cont):

2) $\Sigma(1,2,3,6,7)$

X \ YZ	00	01	11	10
0		1	1	1
1			1	1

$= Y + X'Z$

3) $\Sigma(3,5,6,7)$

X \ YZ	00	01	11	10
0			1	
1		1	1	1

$= XZ + YZ + XY$

Problem #15 (cont):

(5.4)

$$\Sigma(0,2,3,4,6)$$

A \ BC	00	01	11	10
0	1		1	1
1	1			1

$$= C' + A'B$$

Problem #16:

(6.1) $xy + x'y'z' + x'yz'$

$x \backslash yz$	00	01	11	10
0	1			1
1			1	1

$= x'z' + xy$

Problem #16 (cont):

2) $X'Y' + YZ + X'YZ'$

X \ YZ	00	01	11	10
0	1	1	1	1
1			1	

$= X' + YZ$

3) $A'B + BC' + B'C'$

A \ BC	00	01	11	10
0	1		1	1
1	1			1

$= C' + A'B$

Problem #17:

(7.1) $F(A,B,C,D) = \Sigma(4,6,7,15)$

AB \ CD	00	01	11	10
00				
01	1		1	1
11			1	
10				

$= A'BD' + BCD$

7.2) $F(w,x,y,z) = \Sigma(2,3,12,13,14,15)$

wx \ yz	00	01	11	10
00			1	1
01				
11	1	1	1	1
10				

$= wx + w'x'y$

Problem #17 (cont):

3) $F(A, B, C, D) = \Sigma(3, 7, 11, 13, 14, 15)$

AB \ CD	00	01	11	10
00			1	
01			1	
11		1	1	1
10			1	

$= \underline{CD + ABD + ABC}$

Problem #18:

1) $F(w,x,y,z) = \sum(1,4,5,6,12,14,15)$

wx \ yz	00	01	11	10
00		1		
01	1	1		1
11	1		1	1
10				

$= xz' + w'y'z + wxy$

Problem #18 (cont):

2) $F(A,B,C,D) = \sum(0,1,2,4,5,7,11,15)$

AB \ CD	00	01	11	10
00	1	1		1
01	1	1	1	
11			1	
10			1	

$= A'C' + A'B'D' + A'BD + ACD$

3) $F(w,x,y,z) = \sum(2,3,10,11,12,13,14,15)$

wx \ yz	00	01	11	10
00			1	1
01				
11	1	1	1	1
10			1	1

$= wx + x'y$

Problem #18 (cont):

8,4) $F(A,B,C,D) = \Sigma(0,2,4,5,6,7,8,10,13,15)$

AB \ CD	00	01	11	10
00	1			1
01	1	1	1	1
11		1	1	
10	1			1

$= B'D' + A'B + BD$

Problem #19:

.1) $w'z + xz + x'y + wx'z$

wx \ yz	00	01	11	10
00		1	1	1
01		1	1	
11		1	1	
10		1	1	1

$= z + x'y$

Problem #19 (cont):

2) $B'D + A'BC' + AB'C + ABC'$

	00	01	11	10
00		1	1	
01	1	1		
11	1	1		
10		1	1	1

$= BC' + B'D + AB'C$

3) $AB'C + B'C'D' + BCD + ACD' + A'B'C + A'BC'D$

AB \ CD	00	01	11	10
00	1		1	1
01		1	1	
11			1	1
10	1		1	1

$= B'D' + AC + B'C + A'BD \rightarrow$

Problem #19 (cont):

.3 CONT)

OR: $B'D' + AC + CD + A'BD$

4) $wxy + yz + xy'z + x'y$

wx \ yz	00	01	11	10
00			1	1
01		1	1	
11		1	1	1
10			1	1

$= xz + wy + x'y$

Problem #20:

10.1) $XY + YZ + XY'Z$

X \ YZ	00	01	11	10
0			1	
1		1	1	1

$$= \sum (3, 5, 6, 7)$$

2) $C'D + ABC' + ABD' + A'B'D$

AB \ CD	00	01	11	10
00		1	1	
01		1		
11	1	1		1
10		1		

$$= \sum (1, 3, 5, 9, 12, 13, 14)$$

Problem #20 (cont):

0.3) $wxy + x'z' + w'xz$

wx \ yz	00	01	11	10
00	1			1
01		1	1	
11			1	1
10	1			1

$$= \sum (0,2,5,7,8,10,14,15)$$